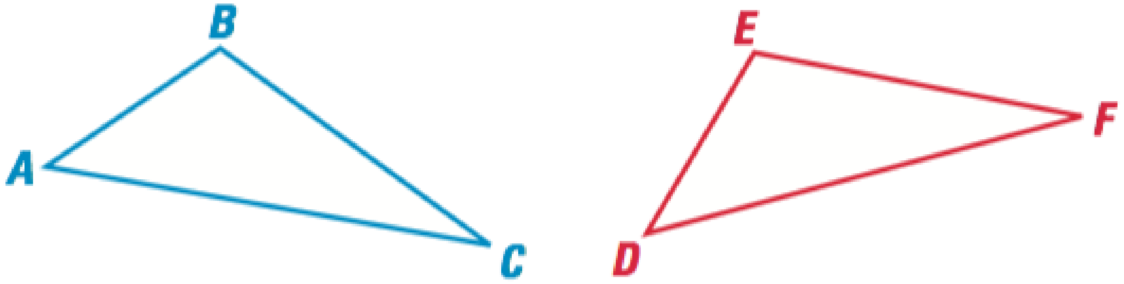
# Chapter 4

# Congruent Triangles

# Section 3
# Proving Triangles are Congruent: SSS and SAS

# GOAL 1: SSS and SAS Congruence Postulates

How much do you need to know about two triangles to prove that they are congruent? In Lesson 4.2, you learned that if all six pairs of corresponding parts (sides and angles) are congruent, then the triangles are congruent.




**If** → **Sides are congruent** **and** **Angles are congruent** **then** → **Triangles are congruent**

| Sides are congruent | Angles are congruent | Triangles are congruent |
|---|---|---|
| **1.** $\overline{AB} \cong \overline{DE}$ | **4.** $\angle A \cong \angle D$ | $\triangle ABC \cong \triangle DEF$ |
| **2.** $\overline{BC} \cong \overline{EF}$ | **5.** $\angle B \cong \angle E$ | |
| **3.** $\overline{AC} \cong \overline{DF}$ | **6.** $\angle C \cong \angle F$ | |

In this lesson and the next, you will learn that you do not need all six of the pieces of information above to prove that the triangles are congruent. For example, if all three pairs of corresponding sides are congruent, then the *SSS Congruence Postulate* guarantees that the triangles are congruent.

## POSTULATE

**POSTULATE 19** ***Side-Side-Side (SSS) Congruence Postulate***

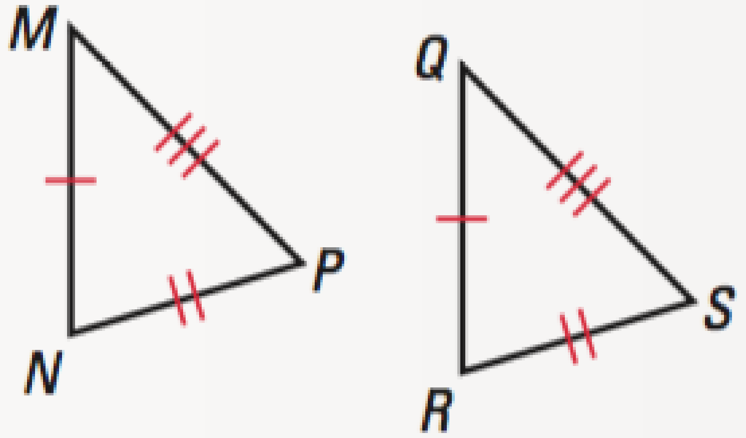
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{MN} \cong \overline{QR}$,

**S**ide $\overline{NP} \cong \overline{RS}$, and

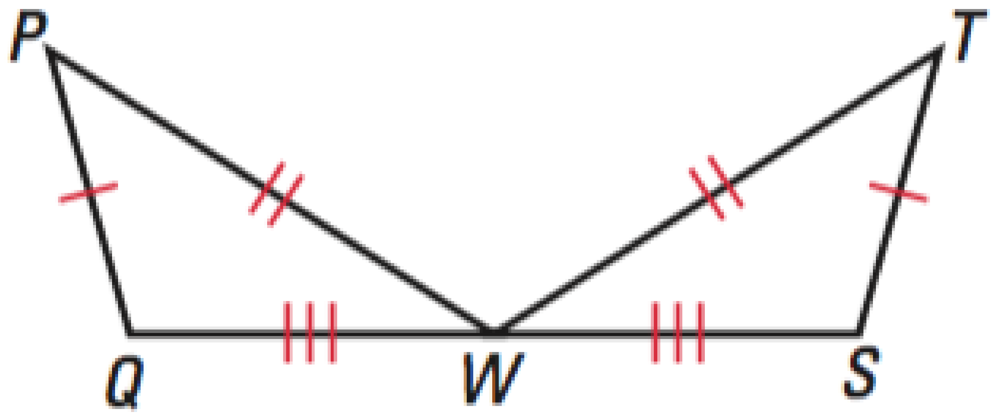
**S**ide $\overline{PM} \cong \overline{SQ}$,

then $\triangle MNP \cong \triangle QRS$.



# Example 1: Using the SSS Congruence Postulate

Prove that $\Delta PQW \cong \Delta TSW$.




PQ cong. TS (given – S)

PW cong. TW (given – S)

QW cong. SW (given – S)

→ Tri. PQW cong. Tri. TSW by SSS

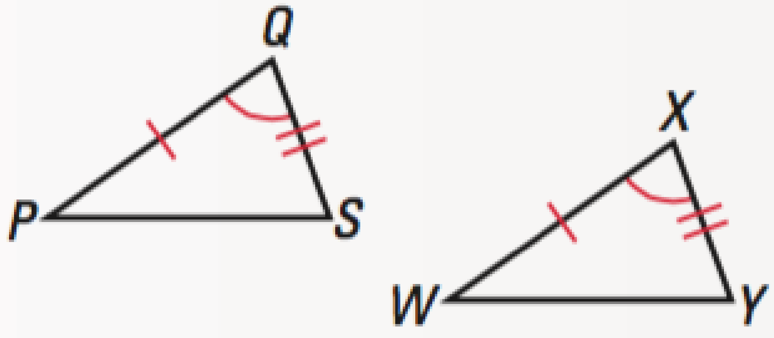
The SSS Congruence Postulate is a shortcut for proving two triangles are congruent without using all six pairs of corresponding parts. The postulate below is a shortcut that uses two sides and that angle that is included between the sides.

**POSTULATE**

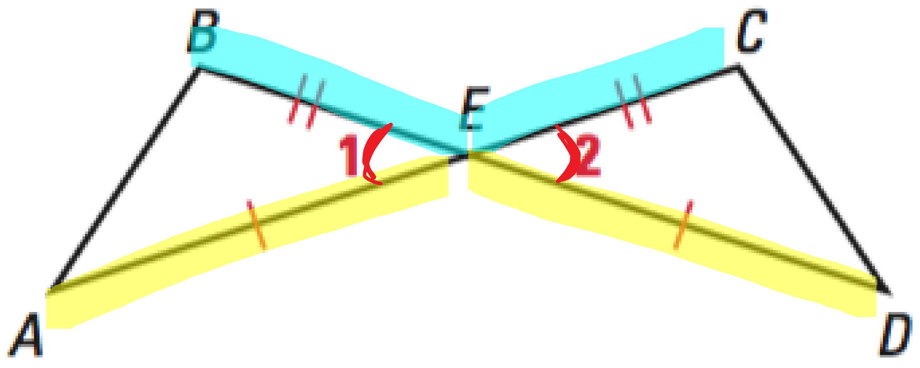
**POSTULATE 20** ***Side-Angle-Side (SAS) Congruence Postulate***

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

| | | |
|---|---|---|
| If | Side | $\overline{PQ} \cong \overline{WX}$, |
| | Angle | $\angle Q \cong \angle X$, and |
| | Side | $\overline{QS} \cong \overline{XY}$, |
| then | | $\triangle PQS \cong \triangle WXY$. |



Example 2: Using the SAS Congruence Postulate




Prove that $\Delta AEB \cong \Delta DEC$.

Statements | Reasons
--- | ---

| Statements | Reasons |
| --- | --- |
| 1) BE cong. CE; AE cong. DE | 1) Given |
| 2) <1 cong. <2 | 2) Vertical <s |
| 3) Tri. AEB cong. Tri. DEC | 3) SAS |

S S A

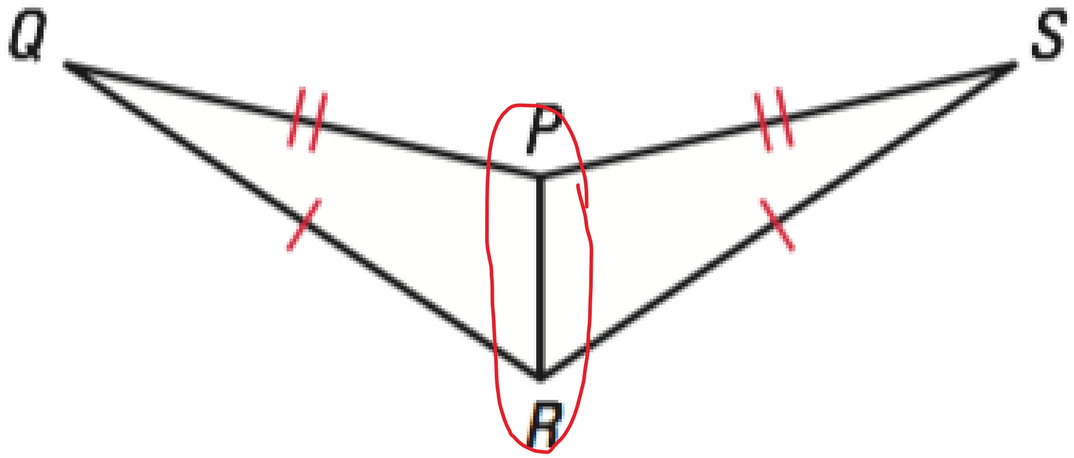
**look for:

Overlapping sides; vertical angles; parallel lines → alt. int. <s, corr. <s, etc.

GOAL 2: Modeling a Real-life Situation

Example 3: Choosing Which Congruence Postulate to Use

Decide whether enough information is given in the diagram to prove that $\Delta PQR \cong \Delta PSR$. If there is enough information, state the congruence postulate you would use.




QP cong. SP (given – S)

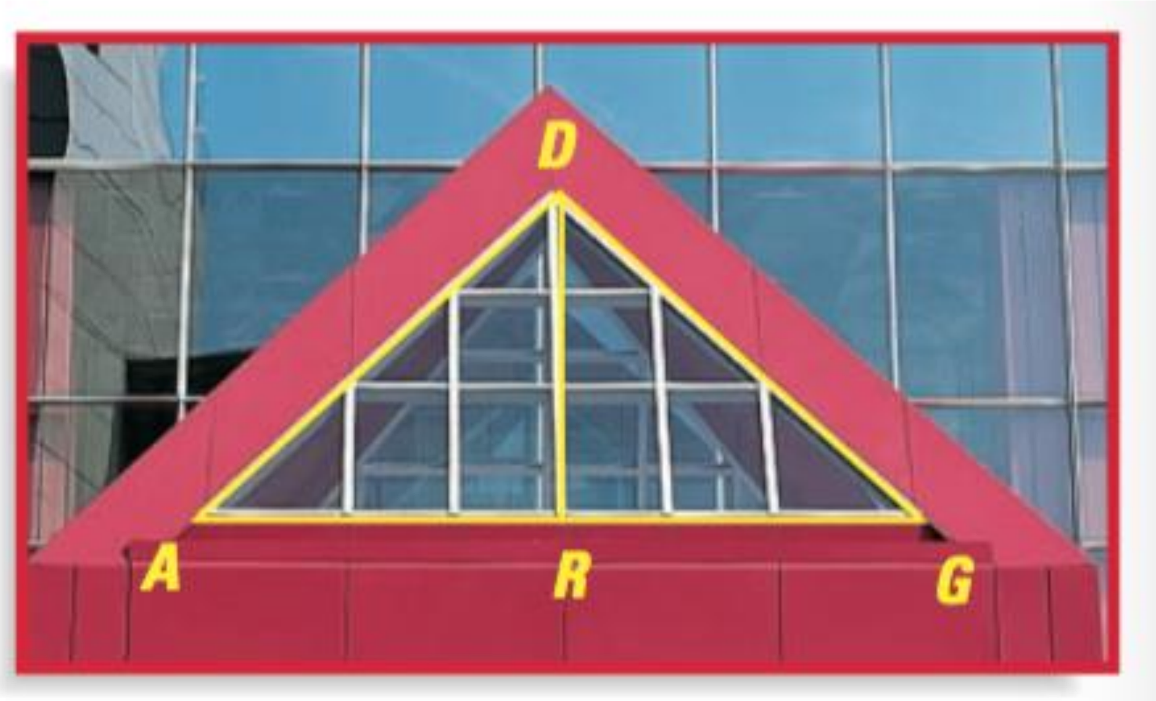
QR cong. SR (given – S)

RP cong. RP (reflexive/overlapping sides – S)

→ Tri. PQR cong. Tri. PSR by SSS

# Example 4: Proving Triangles Congruent

**ARCHITECTURE** You are designing the window shown in the photo. You want to make $\triangle DRA$ congruent to $\triangle DRG$. You design the window so that $\overline{DR} \perp \overline{AG}$ and $\overline{RA} \cong \overline{RG}$. Can you conclude that $\triangle DRA \cong \triangle DRG$?

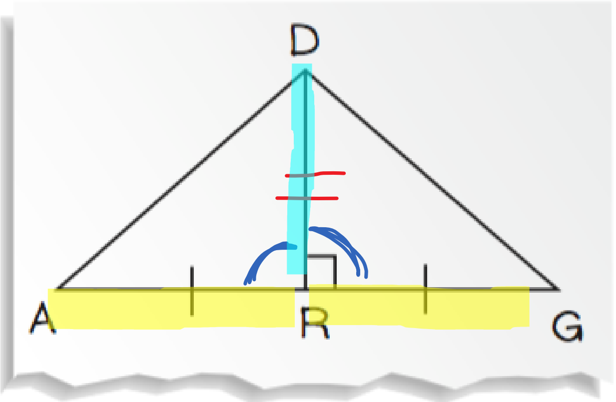


**GIVEN** ▶ $\overline{DR} \perp \overline{AG}$,
$\overline{RA} \cong \overline{RG}$

**PROVE** ▶ $\triangle DRA \cong \triangle DRG$

~~SSS~~

SAS




| Statements | Reasons |
|---|---|
| 1) DR perp. AG; RA cong. RG S | 1) Given |
| 2) <DRA & DRG are right <s | 2) Def. of right <s |
| 3) <DRA cong. <DRG A | 3) Right < Congruence Theorem |
| 4) DR cong. DR S | 4) reflexive/overlapping sides |
| 5) Tri. DRA cong. Tri. DRG | 5) SAS |

# Example 6: Congruent Triangles in a Coordinate Plane

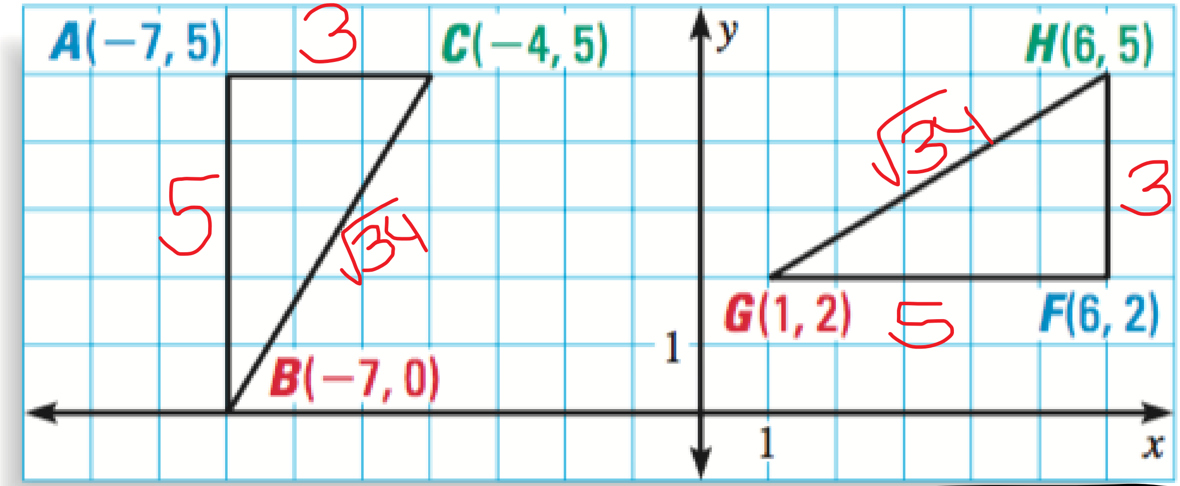
Use the SSS Congruence Postulate to show that $\Delta ABC \cong \Delta FGH$.

AC cong. FH

BC cong. GH

AB cong. FG

→ Tri. ABC cong. Tri. FGH




$$BC \rightarrow \sqrt{(-4 - -7)^2 + (5-0)^2}$$
$$\sqrt{3^2+5^2}$$
$$\sqrt{9+25}$$
$$\sqrt{34}$$

$$GH \rightarrow \sqrt{(6-1)^2+(5-2)^2}$$
$$\sqrt{5^2+3^2}$$
$$\sqrt{25+9}$$
$$\sqrt{34}$$

# EXIT SLIP